The Kerala School of Mathematics

A. Raghuram

Fordham University at Lincoln Center, New York

December 28, 2023

Framework of this talk

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- Introduce the Kerala School of Mathematics (KSM).

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My talk is based on P.P.Divakaran's ICTS lectures, and George Gheverghese Joseph's books.

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Modern Period (1900 – to date)

Ramanujan till the present date.

The Eurocentric History of Calculus

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The *eurocentric* view of the history of Calculus takes for granted that no significant developments took place between the time of Archimedes and the 17th century.

Recognition of the Kerala School of Mathematics - I

The earliest mention of the Kerala School of Mathematics was by **Charles Whish** (1794-1833) in a lecture given in 1832 to the Royal Asiatic Society in London. There was no mention of Madhava. The description of the mathematics was using the language of Newton (e.g., quadrature, fluxion, etc.)

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Whish was in the middle of 15 siblings; he excelled in Persian and Hindustani; worked for East India company; wrote the first book on Malayalam grammar.

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"It seems fair to me to compare Madhava with Newton or Leibniz."
– David Mumford (1974, Fields Medal).

Astronomy – Astrology – Mathematics

There was an older school of Astronomy in Kerala ('Muziris'). They had precisely predicted a solar eclipse of 866 AD. Mathematics at that time evolved out of astronomy. Muziri is a major ancient port city with a vibrant martime exchange with Persia, middle east, north africa, and mediterranean.

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Madhava was as much an astronomer as he was a mathematician. He computed the value of π up to 11 decimal places.

For astronomical calculations (at that time) one does not need the value of π to so many decimals.

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(Sangama in Sanskrit = Kuda in Malayalam = Confluence in English.)

(Grama in Sanskrit = Ur in Malayalam = Town in English.)

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Casually conjectured that Nambutiri brahmin mathematicians were not first born.

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Computational scheme for planetary motion more efficient than Tycho Brahe. Offered a planetary model in which Mercury, Venus, Mars, Jupiter, and Saturn move in eccentric orbits around the sun, which in turn goes around the earth.

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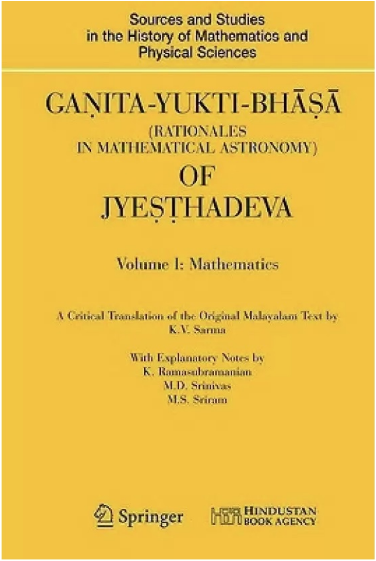
This is what the modern translation of Yuktibhasa looks like:

Yuktibhasa, cont.



Major mathematical ideas contributed by the KSM

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- Sum of a geometric series:

$$1 + x + x^2 + x^3 + \ldots \; = \; \frac{1}{1-x}, \quad |x| < 1.$$

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- Sum of a geometric series:

$$1 + x + x^2 + x^3 + \ldots \; = \; \frac{1}{1-x}, \quad |x| < 1.$$

- Proof by induction of the statement:

$$\lim_{n \to \infty} \frac{1^k + 2^k + \cdots + n^k}{n^{k+1}} \; = \; \frac{1}{k+1}$$

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$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

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- Approximation formulas (special cases of Taylor expansions):

$$\sin(x+h) \approx \sin(x) + h\cos(x) - \frac{h^2}{2}\sin(x).$$

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- A discrete version of the fundamental theorem of Calculus.

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- Many rational approximations to π; conjectured the irrationality of π.

Some important ideas of Calculus missed by the KSM

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- Calculus on a general curve.
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Mathematics: Madhava(-Gregory-Leibniz) Series

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which gives the famous formula:

Theorem (Madhava Series)

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(This uses in effect that $\tan(\pi/6) = 1/\sqrt{3}$.)
He took $d = 9\times 10^{11}$ and computed $C \approx 2827433388233$.
This implies $\pi \approx 3.14159265359$.

$\pi = 3.14159265358979323846264338327950288 4197\ldots$

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- *The man who invented Calculus*, P.P. Divakaran, a mini-course of three lectures titled 'Madhava Lectures', delivered at ICTS, Bengaluru, 2020. `https://www.youtube.com/watch?v=yWZ15EKElH0`

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Thank You!