Brascamp–Lieb Inequalities for Nonabelian Groups

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This is partly a survey, and partly an account of work in progress, joint with various people, especially Ji Li and Chong-Wei Liang.

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Then I consider reasons why some potential inequalities cannot hold.

The inequalities

Fix Haar measures on locally compact groups G and G_j, and take homomorphisms $\sigma_j : G \to G_j$ and $p_j \in [1, +\infty]$; here $j = 1, \dots, J$.

Given functions f_j on G_j, all $f_j \circ \sigma_j$ and $\prod_j f_j \circ \sigma_j$ are functions on G.

The Brascamp–Lieb inequality is:

$$\left| \int_G \prod_j f_j \circ \sigma_j(x) \, \mathrm{d}x \right| \leq C \prod_j \|f_j\|_{L^{p_j}(G_j)} \quad \forall f_j \in L^{p_j}(G_j).$$

We could suppose that the "input functions" f_j are $[0, +\infty]$-valued. The product function on the LHS must be integrable.

The smallest C is the *Brascamp–Lieb constant* $\mathrm{BL}(G, \boldsymbol{\sigma}, \boldsymbol{p})$, where $\boldsymbol{\sigma}$ and $\boldsymbol{p}$ denote $(\sigma_1, \dots, \sigma_J)$ and $(p_1, \dots, p_J)$.

Hölder's inequality

Suppose that $G_j = G$ and each σ_j is the identity map. Then the Brascamp–Lieb inequality becomes Hölder's inequality:

$$\left| \int_G f_1(x) \dots f_J(x) \, \mathrm{d}x \right| \leq C \, \|f_1\|_{L^{p_1}(G)} \cdots \|f_J\|_{L^{p_J}(G)} \, ,$$

which holds when $1/p_1 + \cdots + 1/p_J = 1$. We may take $C = 1$.

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Interpolation between "easy" cases, where the indices are 1 or ∞, proves Hölder's inequality, and gives the best constant.

This is a trivial inequality, in the sense that the structure of σ_j is irrelevant: all that matters is that σ_j preserves measures.

Convolution

The convolution of functions f and g on a locally compact group G is given by

$$f * g(x) = \int_G f(y)\, g(y^{-1}x)\, \mathrm{d}y \qquad \forall x \in G.$$

Young showed that, when $1/r = 1/p + 1/q - 1$,

$$\|f * g\|_{L^r(G)} \leq C \, \|f\|_{L^p(G)} \, \|g\|_{L^q(G)} \, .$$

There are multilinear versions, for instance, we could estimate the norm of $f * g * h$.

Young's convolution inequality

Let H be a locally compact group, let G be $H \times H \times H$ and G_j be H, and define $\sigma_j : G \to G_j$ by

$$\sigma_1(x_1, x_2, x_3) = x_1 \qquad \sigma_2(x_1, x_2, x_3) = x_1^{-1}x_2$$
$$\sigma_3(x_1, x_2, x_3) = x_2^{-1}x_3 \qquad \sigma_4(x_1, x_2, x_3) = x_3.$$

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Then

$$\iiint_G \prod_j f_j(\sigma_j(y))\, \mathrm{d}y_1\, \mathrm{d}y_2\, \mathrm{d}y_3 = \int_H f_1 * f_2 * f_3(y_3) f_4(y_3)\, \mathrm{d}y_3.$$

By Hölder's inequality, $\mathrm{BL}(G, \boldsymbol{\sigma}, \boldsymbol{p}) < \infty$ if and only if

$$\|f_1 * f_2 * f_3\|_{L^{p_4'}(H)} \le C \|f_1\|_{L^{p_1}(H)} \|f_2\|_{L^{p_2}(H)} \|f_3\|_{L^{p_3}(H)},$$

which is the trilinear form of Young's inequality for convolution.

Young's convolution inequality

Interpolation between the cases where three of p_1, p_2, p_3 and p_4 are 1 and the other is 0 proves the trilinear form of Young's inequality, but does not always give the best constant.

The Loomis–Whitney inequality

Let G be a locally compact group and $J \geq 2$. Take $G = H^J$ and $G_j = H^{J-1}$ for all j; let $\sigma_j : G \to G_j$ be the homomorphism that "forgets" the jth coordinate. Loomis and Whitney proved that

$$\left| \int_G \prod_j f_j \circ \sigma_j(x) \, \mathrm{d}x \right| \leq \prod_j \|f_j\|_{L^{J-1}(G_j)} .$$

There is only one possible choice of indices p_j for which this inequality holds. Interpolation is not possible.

Some history

Bennett, Carbery, Christ and Tao (2007) considered the case where $G = \mathbb{R}^n$ and all $G_j = \mathbb{R}^{n_j}$. To avoid degenerate cases, they require that $\bigcap_j \ker \sigma_j = \{0\}$ and that the σ_j are surjective.

They showed that $\mathrm{BL}(G, \boldsymbol{\sigma}, \boldsymbol{p}) < \infty$ if and only if

$$\dim(G) = \sum_j \dim(\sigma_j(G))/p_j \qquad \text{(scaling)}$$

and

$$\dim(V) \leq \sum_j \dim(\sigma_j(V))/p_j \qquad \text{(BCCT)}$$

for all subspaces $V \subseteq G$. There are many subspaces to check!

Lieb's theorem

Theorem (Lieb 1990)

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For a centred gaussian γ,

$$\int_{\mathbb{R}^m} \gamma(x)\,\mathrm{d}x = \det(A)^{1/2}.$$

Corollary to Lieb's theorem

It follows that

$$\mathrm{BL}(G, \boldsymbol{\sigma}, \boldsymbol{p}) = \sup_{\boldsymbol{A}} \frac{\prod_j (\det A_j)^{1/(2p_j)}}{\det \left(\sum_j \sigma_j{}^\top A_j \sigma_j / p_j \right)^{1/2}},$$

where the supremum is taken over all positive definite linear transformations A_j on G_j, where $1 \le j \le J$.

One can work with this expression, as $\boldsymbol{A}$ is finite dimensional.

Why generalise this?

These inequalities are applied in harmonic analysis and partial differential equations. It is hoped that they may shed some light on a circle of problems including restriction in harmonic analysis, the Kakeya problem in geometry, and the Mizohata–Takeuchi conjecture in PDE.

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Computer scientists are interested in versions of these inequalities where the linear maps $\sigma_j : \mathbb{R}^n \to \mathbb{R}^{n_j}$ are replaced by homomorphisms of finitely generated discrete abelian groups $\sigma_j : G \to G_j$. This discrete version is connected to one of Hilbert's problems.

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Bennett and Jeong found the corresponding result when G and all G_j are products of tori and finite groups.

Finally, Bennett and I are finalising a study for general LCA groups.

The nonabelian case

We have begun to think about the nonabelian case. We may and shall suppose that the $\sigma_j : G \to G_j$ are canonical projections of G onto G/N_j, where $N_j = \bar{N}_j \trianglelefteq G$, and $\bigcap_j \ker \sigma_j = \{e\}$.

We consider two examples, that suggest that this theory may be less interesting.

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We consider two examples, that suggest that this theory may be less interesting.

The Heisenberg group $\mathbb{H}^n$ and the $ax + b$ group G_n are the groups of all real matrices of the form

$$g(x, y, z) = \begin{pmatrix} 1 & x & z \\ 0 & I_n & y \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad g(a, b) = \begin{pmatrix} a & b \\ 0 & I_n \end{pmatrix}$$

respectively. These are block matrices: x and b are row vectors with n entries and y is a column vector with n entries, and I_n is the $n \times n$ identity matrix. We usually take $a \in \mathbb{R}^+$.

The case of the Heisenberg group

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We need two preliminary results. First we recall a simple variant of Kronecker's approximation theorem.

Theorem

Given finitely many real numbers $\alpha_1, \cdots, \alpha_J$ and arbitrarily small $\epsilon > 0$, there exists a sequence $\{t_m\}_m$ such that $t_m \to \infty$ when $m \to \infty$ and, for all m and j, there exists an integer $k_{j,m}$ such that

$$|t_m - \alpha_j k_{j,m}| < \epsilon \qquad \forall j \in \{1, \dots, J\} \quad \forall m \in \mathbb{N}.$$

The case of the Heisenberg group.2

Next, the centre of $\mathbb{H}^n$ is the subgroup $Z = \{g : x = 0, y = 0\}$.

Lemma

Suppose that N is a normal subgroup of the Heisenberg group $\mathbb{H}^n$. Then $N \subseteq Z$ or $N \supseteq Z$.

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Lemma

Suppose that N is a normal subgroup of the Heisenberg group $\mathbb{H}^n$. Then $N \subseteq Z$ or $N \supseteq Z$.

Proof. If $g = g(x, y, z) \in N$, then $g^{-1}(g')^{-1}gg' \in N$ for all $g' = g(x', y', z') \in \mathbb{H}^n$. There is a bilinear map B such that

$$g^{-1}(g')^{-1}gg' = g(0, 0, B(x, x', y, y')).$$

If $N \not\subseteq (0, \mathbb{R})$, then there exists $g \in N$ such that $x \neq 0$ or $y \neq 0$; in this case, the commutator $g^{-1}(g')^{-1}gg'$ varies over Z as g' varies over $\mathbb{H}^n$, and $N \supseteq Z$. Otherwise $N \subseteq Z$, as required. $\square$

The case of the Heisenberg group. 3

A homogeneous group G is Heisenberg-like if it has a central subgroup Z, isomorphic to $\mathbb{R}$, with the property that every normal subgroup of G is either contained in Z or contains Z.

The groups of $n \times n$ upper triangular unipotent matrices are Heisenberg-like when $n \geq 3$.

Proposition

The only Brascamp–Lieb inequalities in which G is a Heisenberg-like group are multilinear Hölder inequalities.

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Proof. The subgroup Z is isomorphic to $\mathbb{R}$. We write $(0, t)$ for the element of Z that corresponds to $t \in \mathbb{R}$. The closed subgroups of Z are all of the form $(0, \mathbb{Z}\alpha)$ for some $(0, \alpha) \in Z$.

The case of the Heisenberg group. 4

Assume that $\mathrm{BL}(G, \boldsymbol{\sigma}, \boldsymbol{p}) < \infty$.

Each σ_j is the canonical projection of G onto a group $G_j = G/N_j$, where N_j is either $\{e\}$, or of the form $(0, \alpha_j\mathbb{Z})$, or contains $(0, \mathbb{R})$. By renumbering the σ_j if necessary, we may and shall suppose that $\ker \sigma_j = \{e\}$ when $j = 1, \ldots, I$. If there are no such σ_j, then $I = 0$.

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For $j \leq I$, the group G_j is isomorphic to G. Now $f_j \in L^{p_j}(G_j)$ if and only if $f_j \circ \sigma_j \in L^{p_j}(G)$ and

$$\|f_j \circ \sigma_j\|_{L^{p_j}(G)} = C_j \|f_j\|_{L^{p_j}(G_j)}.$$

By Hölder's inequality, if $1/p = 1/p_1 + \cdots + 1/p_I$, then $\prod_{j \leq I} f_j \circ \sigma_j \in L^p(G)$, and every function in $L^p(G)$ arises in this way. Since $\prod_j f_j \circ \sigma_j$ is integrable, $p \geq 1$.

The case of the Heisenberg group. 5

By the converse of Hölder's inequality and taking powers,

$$\int_G \prod_j f_j \circ \sigma_j(x)\,\mathrm{d}x \leq C \prod_j \|f_j\|_{L^{p_j}(G_j)}$$

$\iff$

$$\left(\int_G \left|\prod_{j>I} f_j \circ \sigma_j(x)\right|^{p'} \mathrm{d}x\right)^{1/p'} \leq C' \prod_{j\geq I} \|f_j\|_{L^{p_j}(G_j)},$$

$\iff$

$$\int_G \left|\prod_{j>I} f_j \circ \sigma_j(x)\right| \mathrm{d}x \leq C' \prod_{j>I} \left(\int_{G_j} |f_j(x)|^{p_j/p'}\,\mathrm{d}x\right)^{p'/p_j}$$

for all $f_j \in L^{p_j/p}(G_j)$; that is, $\mathrm{BL}(G, \boldsymbol{\rho}, \boldsymbol{q}) \leq C'$, where $\boldsymbol{\rho} = (\sigma_{I+1}, \ldots, \sigma_J)$ and $\boldsymbol{q} = (p_{I+1}/p', \ldots, p_J/p')$.

The case of the Heisenberg group. 6

If $J = I$, then we have a multilinear Hölder inequality.

Otherwise $J > I$ and we derive a contradiction.

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Otherwise $J > I$ and we derive a contradiction.

There are homomorphisms $\sigma_{I+1}, \ldots, \sigma_J$, all of whose kernels are nontrivial, and for each $j > I$, there exists $\alpha_j \in \mathbb{R}^+$ such that $(0, \alpha_j \mathbb{Z}) \subseteq \ker \sigma_j$. We take a nonempty relatively compact open set U in G, and nonnegative functions $f_j \in C_c(G_j)$ such that $f_j(y) = 1$ for all $y \in (0, [-\varepsilon, \varepsilon])U$.

By Kronecker's Theorem, there is an infinite sequence of elements $(0, t_m)$ of Z such that $t_m \to \infty$ as $m \to \infty$ and, for all m and j, there exist $k_{j,m} \in \mathbb{Z}$ such that $|t'_m| < \varepsilon$, where $t'_m = t_m - \alpha_j k_{j,m}$. By passing to a subsequence if necessary, we may suppose that the sets $(0, t_n)^{-1} U$ are disjoint.

The case of the Heisenberg group. 7

Since $(0, \alpha_j m) \in \ker(\sigma_j)$ for all $m \in \mathbb{Z}$,

$$\begin{aligned}
\int_G \prod_j f_j \circ \sigma_j(x) \, \mathrm{d}x &\geq \sum_m \int_{(0,t_m)^{-1}U} \prod_j f_j \circ \sigma_j(x) \, \mathrm{d}x \\
&= \sum_m \int_U \prod_j f_j \circ \sigma_j((0, t_m)x) \, \mathrm{d}x \\
&= \sum_m \int_U \prod_j f_j \circ \sigma_j((0, t'_m)x) \, \mathrm{d}x \\
&= \sum_m \int_U 1 \, \mathrm{d}x \\
&= \infty,
\end{aligned}$$

and $\mathrm{BL}(G, \boldsymbol{\rho}, \boldsymbol{q}) = \infty$. Hence $I = J$ and the only Brascamp–Lieb inequality is a multilinear Hölder inequality. $\square$

The $ax + b$ group

Theorem ([C-Li-Liang-Shen])

There are no nontrivial Brascamp–Lieb inequalities where G is the $ax + b$ group.

Proof. We outline the proof.

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First, the closed normal subgroups of G_n are of two forms: either $\{(1, b) : b \in V\}$, where V is a vector subspace of $\mathbb{R}^n$, possibly $\{0\}$ or $\mathbb{R}^n$, or $\{(a^m, b) : m \in \mathbb{Z}, b \in \mathbb{R}^n\}$, where $a \neq 1$.

Without loss of generality, the homomorphisms σ_j are all canonical projections of G_n onto G_n/N_j, where N_j is a closed normal subgroup. We renumber the subgroups such that $N_j \leq (1, \mathbb{R}^n)$ when $j \leq I$ and $N_j > (1, \mathbb{R}^n)$ when $j > I$.

When $j \leq I$, G/N_j is a group G_{n_j}, where $0 \leq n_j \leq n$. When $j > I$, G/N_j is compact and we may take f_j to be 1.

The $ax + b$ group. 2

Assume that $\mathrm{BL}(G, \boldsymbol{\sigma}, \boldsymbol{p}) < \infty$. The modular function of G_n is $(a, b) \mapsto a^n$.

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By choosing functions f_j when $j \leq I$ and considering their right translates by $(a, 0)$, we obtain a homogeneity condition:

$$\sum_{j \leq I} \frac{n_j}{p_j} = n.$$

By choosing functions f_j that are products of functions on $(\mathbb{R}^+, 0)$ and functions on $(1, \mathbb{R}^n)$, and translating the first factor only, we can also show that

$$\sum_{j \leq I} \frac{1}{p_j} \leq 1.$$

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By choosing functions f_j that are products of functions on $(\mathbb{R}^+, 0)$ and functions on $(1, \mathbb{R}^n)$, and translating the first factor only, we can also show that

$$\sum_{j \leq I} \frac{1}{p_j} \leq 1.$$

Hence $n_j = n$ for all $j \leq I$, that is, $N_j = \{e\}$.

The $ax + b$ group. 3

The mappings σ_j are isomorphisms when $j \leq I$, and as in the case of Heisenberg-like groups, we can show that $\mathrm{BL}(G_n, \boldsymbol{\rho}, \boldsymbol{q}) < \infty$, where $\boldsymbol{\rho} = (\sigma_{I+1}., \dots, \sigma_J)$ and $\boldsymbol{q} = (p_{I+1}/p', \dots, p_J/p')$.

But this is impossible, because for this range of j all $f_j \circ \sigma_j$ are constant on cosets of $(1, \mathbb{R}^n)$ so their product is also constant on these cosets, and then the product cannot be integrable. $\square$

Summary

We know quite a lot about Brascamp–Lieb constants on general locally compact abelian groups. The next stage is to consider nonabelian groups systematically.

Some examples have already been considered. On the one hand, it might seem that noncommutativity would make proofs more difficult; on the other hand, it also means that there are fewer possible homomorphisms and fewer possible inequalities to consider.

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